determined without reference to the chemical kinetics. Mixing of equilibrium and nonequilibrium equations is valid if the chemical system is treated as two subsystems, one of which is assumed to be in local chemical equilibrium and the other rate-controlled. The systems are coupled through the requirements of element conservation, and the rate-controlled subsystem can be thought of as a source or sink of chemical species which forms a set of boundary conditions for the equilibrium subsystem. If the equilibrium subsystem is assumed to include species 1, . . ., M, the conditions $DG_i/Dt = 0$ yield M equations:

$$\frac{\partial G_k}{\partial \rho}\left(\rho\frac{D\rho}{Dt}\right) + \frac{\partial G_k}{\partial T}\left(\rho\frac{DT}{Dt}\right) = -\sum_{i=1}^{N}\left(\frac{\partial G_k}{\partial Y_i}\right)\left(\rho\frac{DY_i}{Dt}\right) \qquad k = 1, \ldots, M \qquad (2)$$

as shown by Libby. The right-hand side of each equation will involve only Y_k, Y_{N-L+1}, . . ., Y_N. There will be N-L-M equations of the form of Eq. (1) remaining:

$$\rho(DY_i/Dt) = \dot{w}_i \quad i = M+1, \ldots, N\text{-}L \qquad (3)$$

Since none of the σ_j, $j = 1, \ldots, M$ appear in these equations, the right-hand sides will be determinate. Together with the element conservation equations, these form a set of algebraic equations that may be solved for the unknown species derivatives at any point in the flow.

The reason that the mixing of equilibrium and nonequilibrium assumptions must be approached with such care is that the equilibrium limit of the nonequilibrium problem is a singularity. For true chemical equilibrium to exist, as required by the condition $G_j = 0$, there must be no net production or consumption of any chemical species. This is impossible in any system in which finite gradients are present, as in a reacting flow. In computer solutions of reacting flow problems, difficulties are encountered when the forward and reverse reaction rates both are very large and their difference is too small to compute accurately. In such cases the equilibrium assumption that both forward and reverse rates are infinitely large and equal may furnish a good approximation. However, the gradient in species concentration computed from Eq. (2) is not dependent on the known finite rates of any reactions involving species k. Rather it is the rate at which the species concentration must change in order to maintain equilibrium (zero net production) when the density and temperature are changing. Hence the analogy between equilibrium and nonequilibrium is not complete.

Author's Reply to Comment by R. E. Mates

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The present author assumed in the note under discussion[1] that the problem was "well-posed," that is, that the equilibrium conditions corresponding to the "fast" reactions, $G_j \simeq 0$, were indeed all independent; if they are not, the problem is "ill-posed" by definition. Thus the observations in the second paragraph do not appear appropriate.

In addition, the present author does not understand the first two sentences of the third paragraph of the foregoing comment. It is precisely in order "to compute contributions to gradients in species concentrations due to individual reactions," namely, from the "fast" reactions, and because the concentration of a species "involved in local chemical equilibrium with some other species" *cannot* "be determined without reference to the chemical kinetics" of the "slow" reactions that there exists a problem in applying the concept of partial equilibrium in flowing systems, a problem posed and solved by one of perhaps several methods by the present author.

The system described in connection with Eq. (2) of the preceding comment appears to be highly artificial in that the M fast reactions are postulated to involve only M species; in this case, the commentor is correct that no indeterminacies arise and that no problem exists.

With respect to the final paragraph, it is understood generally by fluid mechanicians that the concept of equilibrium chemical behavior in a flowing system involves whatever "net production or consumption of any chemical species" required to maintain at each point in the flow an equilibrium state. The forward and reverse rates corresponding to a single reaction step at equilibrium are relatively large *but not equal*. The difficulties connected with obtaining equilibrium flows from computer programs applicable to finite rate chemistry are well known and, of course, are associated with the size of the reaction rates.

In view of the doubts raised by Mates[1] concerning the present author's treatment of partial equilibrium, it may be appropriate to point out that the procedure in question has been employed to study the heat release phase of the hydrogen-air reaction carried out at constant pressure;[2] there was no difficulty encountered in applying the treatment under discussion.

References

[1] Mates, R. E., "Comments on 'Treatment of partial equilibrium in chemically reacting flows'," AIAA J. 1, 723–724 (1963).

[2] Ferri, A., Libby, P. A., and Zakkay, V., "Theoretical and experimental investigation of supersonic combustion," Polytech. Inst. of Brooklyn, Aerodynamics Lab., PIBAL Rept. 713, ARL 62-467, AD 291712 (September 1962); also *Proceedings of the Third International Council of Aeronautical Sciences Congress* (to be published).

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Errata: "The Status of Unsteady Aerodynamic Influence Coefficients"

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The derivation of aerodynamic influence coefficients (AICs) from slender-body theory given in Ref. 1, pp. 60–72, has been found to be incorrect. Since the standard NASA stability-axis system is used (with the relative wind in the *negative x* direction), the substantial derivative operator in Eq. (179) et seq. should read $-V(\partial/\partial x) + (\partial/\partial t)$, and, hence, the sign of the velocity should be reversed throughout the derivation. The net result is that the signs of the imaginary parts of all the AIC elements (i.e., the damping terms) are reversed; the signs of the real parts are not affected. The mistake was discovered while programming the equations for digital computation, and the corrected derivation as well as the FORTRAN computer program are reported in Ref. 2.

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In addition to the static and oscillatory cases, Ref. 2 also considers the transient case. The simplicity of slender-body theory permits the definition of a series of transient AICs from which the control-point forces can be found in terms of the control-point deflections and their first two derivatives:

$$\{F(t)\} = (qS/\bar{c})([C_{hs}]\{h\} + [C_{hd}]\{\dot{h}\bar{c}/V\} + [C_{hi}]\{\ddot{h}\bar{c}^2/V^2\})$$

The option for the transient case in the computer program of Ref. 2 generates the static AICs $[C_{hs}]$, the damping AICs $[C_{hd}]$, and the inertial AICs $[C_{hi}]$.

References

[1] Rodden, W. P. and Revell, J. D., "The status of unsteady aerodynamic influence coefficients," Inst. Aerospace Sci. Paper FF-33 (January 1962).

[2] Rodden, W. P., Farkas, E. F., and Takata, G. Y., "Aerodynamic influence coefficients from slender body theory: analytical development and computational procedure," Aerospace Corp. Rept. TDR-169 (3230-11) TN-6 (October 31, 1962).

Comments on "Angle of Attack and Sideslip from Pressure Measurements on a Fixed Hemispherical Nose"

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KEENER[1] calls attention to "a simple method for sensing angle of attack and sideslip that appears to have been overlooked in the design of flow-direction probes." The writer, in a study conducted for the then Wright Air Development Center[2] five years ago, used a simple variant of Keener's method to normalize the pressure difference on blunt angle-of-attack, angle-of-sideslip probes. This involves the use of the pitot pressure alone (or P_{90} in Keener's notation) rather than the difference between the pitot pressure and some other surface pressure. For $M > 3$, this method has the advantage of the requiring fewer measurements and involves a simpler calibration formula. Since

$$P_{90} = P_{90\alpha=0} \cos^2\alpha \tag{1}$$

on a hemisphere for these conditions, the calibration formula becomes

$$\frac{P_l - P_u}{P_{90}} = \frac{\cos^2(\delta_l - \alpha) - \cos^2(\delta_u + \alpha)}{\cos^2\alpha} \tag{2}$$

where δ_l is the angular displacement of the lower orifice measured from the pitot pressure source and δ_u is the angular displacement of the upper source. Equation (2) has the further advantage of being somewhat more linear for $\alpha < 10°$ than Keener's result. When δ_u and δ_l are 45°, for example, Eq. (2) becomes simply

$$(P_l - P_u)/P_{90} = 2 \tan\alpha \tag{3}$$

It may be of interest to note that a similar relation has been found to give good agreement with experimental results for an angle sensor made from a spherically capped cone with a small pitot source in the nose. It also was found that, to account for the change in pressure distribution with change in Mach number, one could replace α by $\alpha/(1 + 1/M^2)$ with generally good results. This may prove to be a more direct method than that suggested by Keener, i.e., generalizing the exponent in Eqs. (1) and (2), if $\alpha > 20°$. Keener's method of using a pressure difference to normalize $p_l - p_u$ apparently makes the result insensitive to changes in M for $1.5 \leq M \leq 3$ and $\alpha < 20°$, which the present method does not.

Finally, it might be pertinent to mention that recent wind-tunnel experience has indicated that Keener's estimate of the accuracy attainable (within ±1°) may be too conservative. With carefully calibrated pressure gages of high quality, data scatter has been kept to $\pm\frac{1}{2}°$ or less in most cases.

References

[1] Keener, E. R., "Angle of attack and sideslip from pressure measurements on a fixed hemispherical nose," J. Aerospace Sci. **29**, 1129–1130 (1962).

[2] Smetana, F. O. and Headley, J. W., "A further study of angle-of-attack, angle-of-sideslip, pitot-static tubes," Wright Air Dev. Center, WADC TR 57-234 (June 1958).

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Blast-Hypersonic Flow Analogy

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IN view of the recently published erratum,[1] the footnote on p. 1342 of Ref. 2 should be disregarded, Eqs. (5-8) of Ref. 2 being correct.

References

[1] Jones, D. L., "Erratum: strong blast waves in spherical, cylindrical and plane shocks," Phys. Fluids **5**, 637 (1962).

[2] Lukasiewicz, J., "Blast-hypersonic flow analogy theory and applications," ARS J. **32**, 1341–1346 (1962).

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Comment on "Heat Transfer in Planetary Atmospheres at Super-Satellite Speeds"

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Nomenclature

h = enthalpy, ft²/sec²
u_1 = velocity at outer edge boundary layer, fps
ρ_w = density, slug/ft³
$\dot{q}_w$ = stagnation point heat transfer rate, Btu/ft²-sec
μ_w = viscosity, slug/sec-ft
β = external velocity gradient, du_1/dx sec^{-1}
Nu = $[\dot{q}_w x c_{p_w}/k_w(h_0 - h_w)]$
Re_x = $u_1 x/\nu_w$

HOSHIZAKI[1] has shown that the dependence of the stagnation point heat transfer rate on flow field properties is the same both at low speeds, on the order of 5 to 10,000 fps,

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